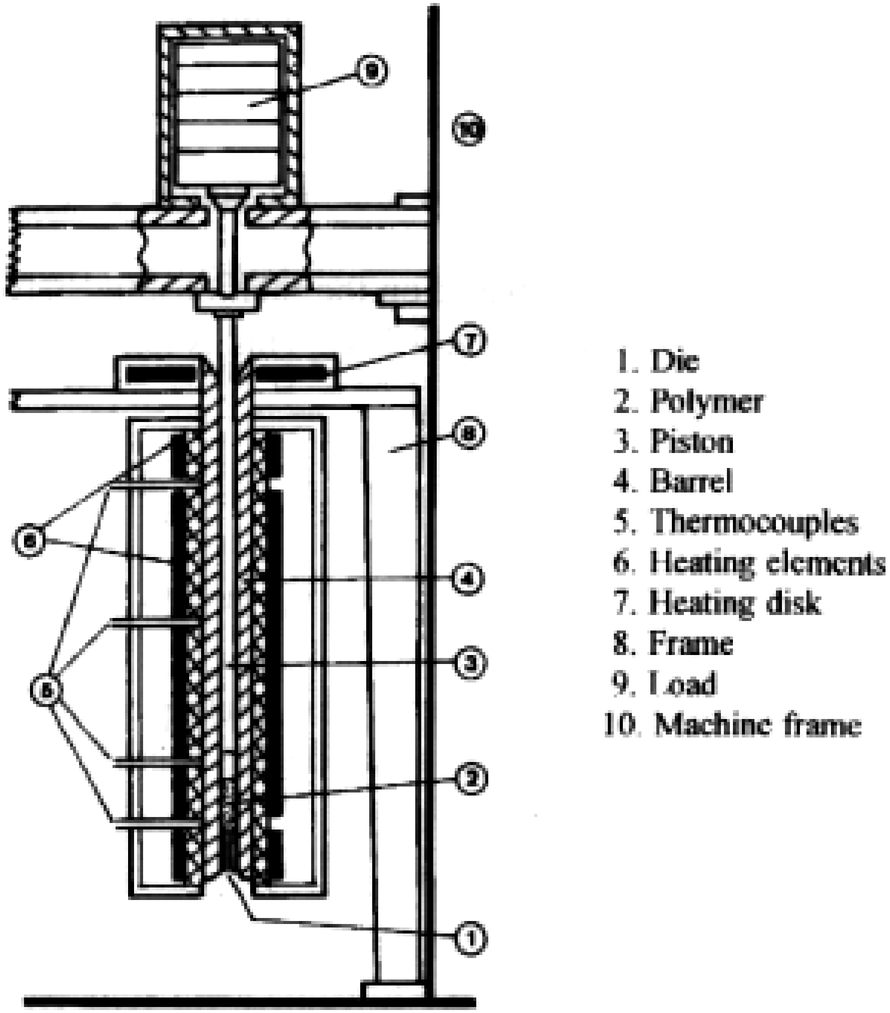
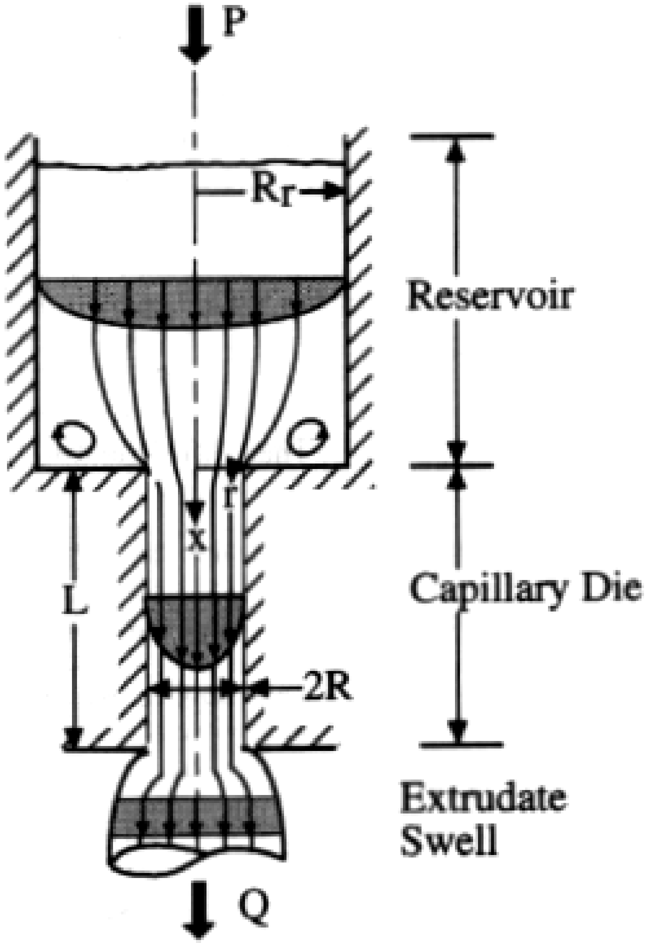
Capillary Rheometer

The capillary rheometer (or viscometer) is the most common device for measuring viscosity. Gravity, compressed gas or a piston is used to generate pressure on the test fluid in a reservoir. A capillary tube of radius R and length L is connected to the bottom of the reservoir. Pressure drop and flow rate through this tube are used to determine viscosity



Capillary Rheometer Analysis

The flow situation inside the capillary rheometer die is essentially identical to the problem of pressure driven flow inside a tube (Poiseuille flow).



- We can record force on piston, F (or the pressure drop ΔP), and volumetric flow rate, Q

Capillary Rheometer Analysis

Recall from Fluid Mechanics

- Shear stress profile inside the tube:

$$\tau = \frac{r}{2}\left(\frac{\Delta P}{L}\right) \qquad \text{At the wall (r=R):} \qquad \tau_W = \frac{R}{2}\left(\frac{\Delta P}{L}\right) \qquad (1)$$

- Velocity profile inside the tube:

$$u(r) = \frac{\Delta P\, R^2}{4\mu L}\left[1-\left(\frac{r}{R}\right)^2\right]$$

- Hagen-Poiseuille law for pressure driven flow of **Newtonian fluids** inside a tube:

$$\Delta P = \mu L \frac{8Q}{\pi} R^{-4}$$

Capillary Rheometer Analysis

- Shear rate:

$$\dot{\gamma} = \frac{du}{dr} = \frac{\Delta P}{2\mu L} R = \frac{4Q}{\pi R^3} = \dot{\gamma}_{apparent}$$

True shear rate for Newtonian fluids but Apparent shear rate ($\dot{\gamma}_{app}$) for non-Newtonian fluids

∴ For non-Newtonian fluids if we use the apparent shear rate then we can only calculate an **Apparent Viscosity**:

$$\eta_{app} = \frac{\tau_w}{\dot{\gamma}_{app}}$$

Capillary Rheometer Analysis

For non-Newtonian fluids the *Rabinowitch analysis* is followed

- From the definition of the volumetric flow rate through a tube:

$$Q = 2\pi\int_0^R r\, u(r)\, dr \xrightarrow{\text{integrating by parts}} Q = \pi r^2 u\Big|_0^R - \int_0^R \pi r^2 \left(\frac{du}{dr}\right) dr$$

- Applying the “no-slip” boundary condition and eliminating r with the aid of eq. (1)

$$\frac{\tau_W^3 Q}{\pi R^3} = \int_0^{\tau_W} \tau^2 \left(\frac{du}{dr}\right) d\tau$$

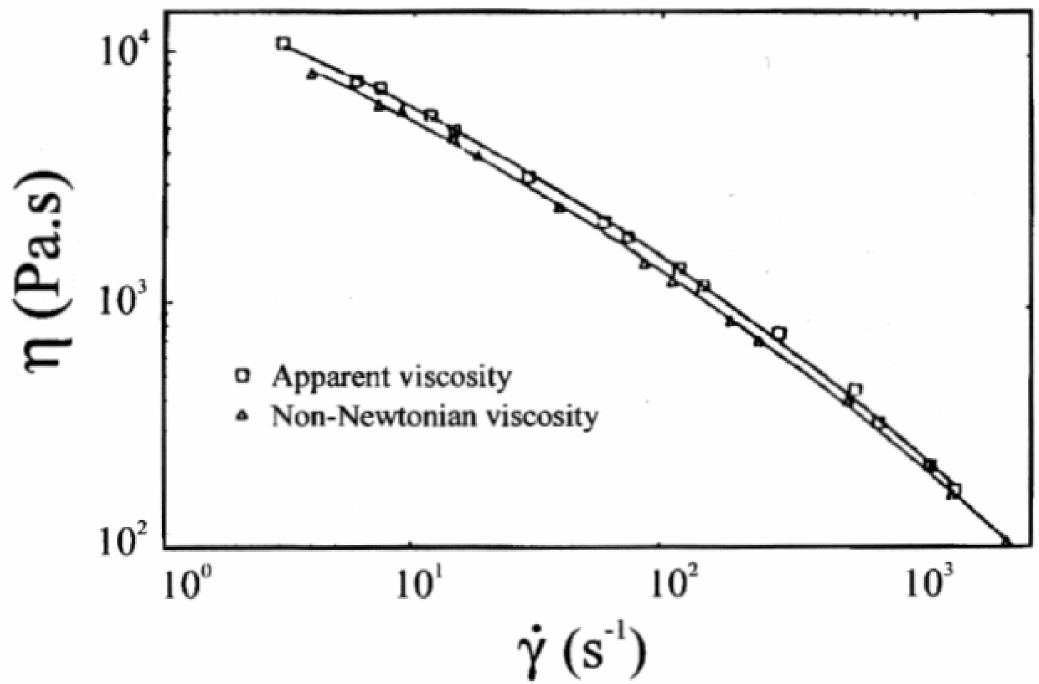
Capillary Rheometer Analysis

After several manipulations we obtain the **Rabinowitch equation**

$$\dot{\gamma}_W = \frac{4Q}{\pi R^3}\left(\frac{3}{4}+\frac{1}{4}\frac{d\ln Q}{d\ln\tau_W}\right) = \dot{\gamma}_{app}\left(\frac{3}{4}+\frac{1}{4}\frac{d\ln Q}{d\ln\tau_W}\right) \qquad (2)$$

∴ The **Real Viscosity** of the polymer melt is:

$$\eta = \frac{\tau_w}{\dot{\gamma}_W}$$



Capillary Rheometer Analysis

- To obtain the “true” shear rate we must plot Q vs τ_w on logarithmic coordinates to evaluate the derivative $d\ln Q/d\ln\tau_w$ for each point of the curve
- For power-law fluids, it turns out that the slope is:

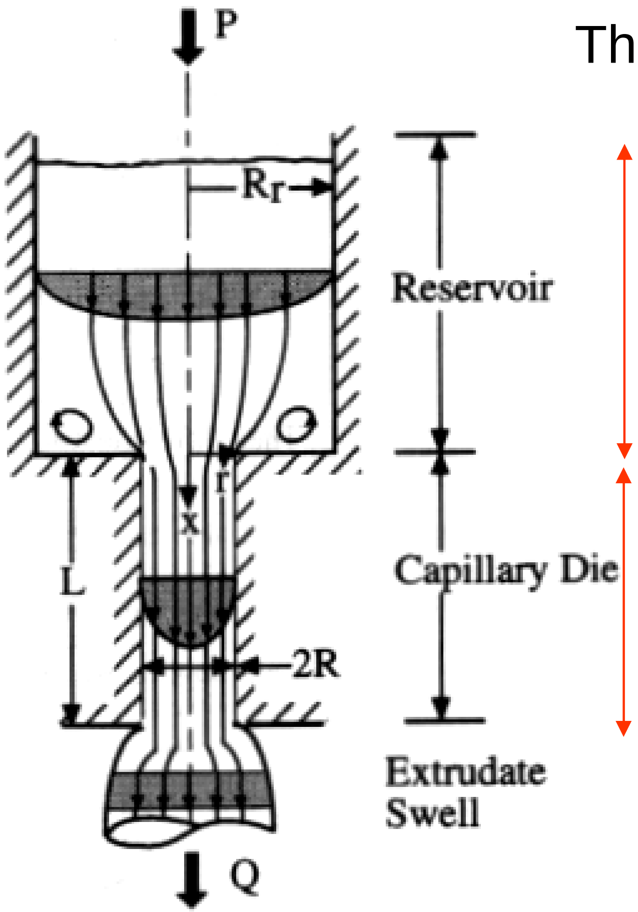
$$\frac{d\ln Q}{d\ln\tau_W} = \frac{1}{n}$$

∴ The Rabinowitch equation becomes:

$$\boxed{\dot{\gamma}_W = \frac{4Q}{\pi R^3}\frac{3n+1}{4n}}$$

Entrance Pressure Drop

➢In the previous analysis we have assumed that the measured ΔP by the instrument corresponds to the pressure drop inside the capillary die, ΔP_{cap}



Therefore from eq. (1) we had: $$\tau_W = \frac{\Delta P}{2\frac{L}{R}}$$

$\Delta P_{res} \sim 0$

$\Delta P_e =$ Entrance Pressure Drop

ΔP_{cap}

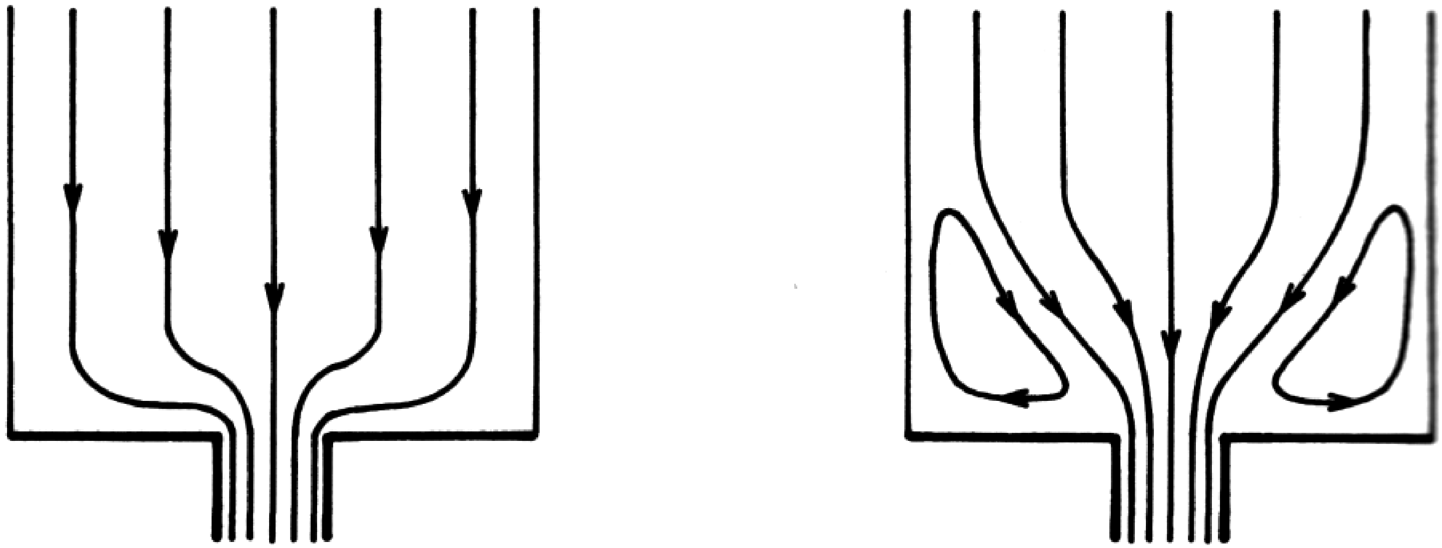
In reality

$$\Delta P_{total} = \Delta P_{res} + \Delta P_e + \Delta P_{cap}$$

Entrance Pressure Drop

Newtonian Fluids and some melts such as HDPE and PP

Fluids with pronounced non-Newtonian behaviour



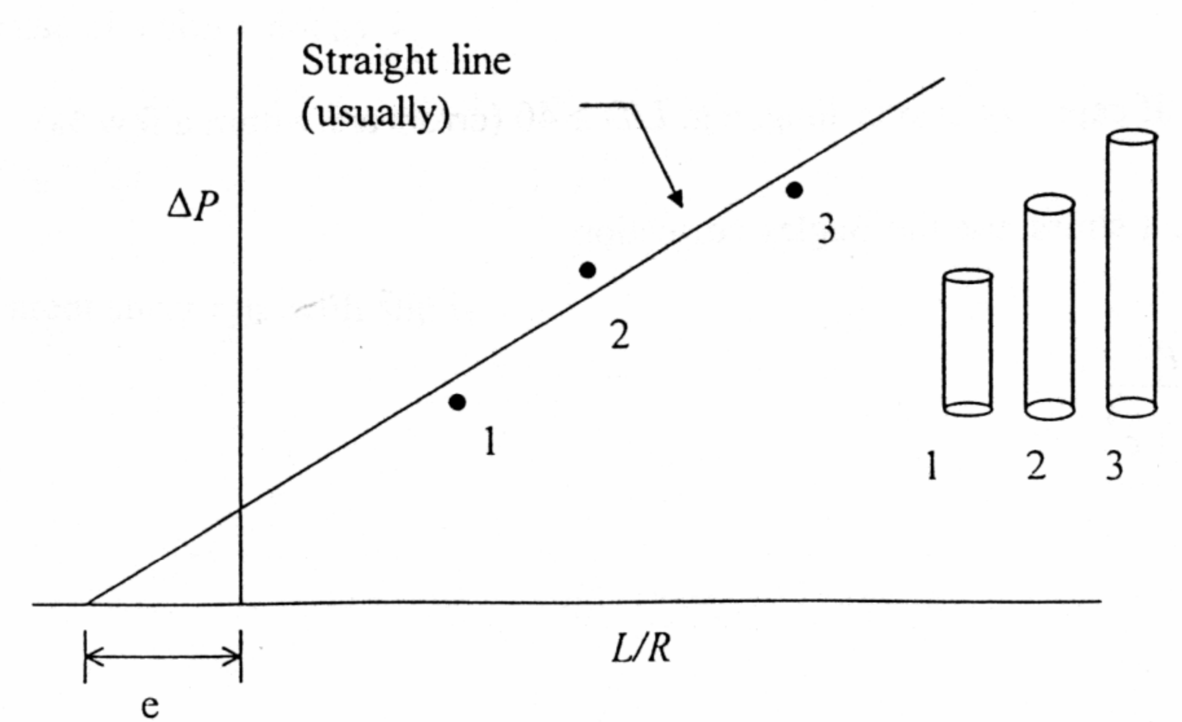
(a) (b)

Bagley Correction for ΔP_e

Unless a very long capillary is used (L/D>100), entrance pressure drop may considerably affect the accuracy of the measurements.

➢The **Bagley correction** is used to correct for this, by assuming that we can represent this extra entrance pressure drop by an equivalent length of die, e:

– Three or four capillaries are used and results are plotted as DP vs L/R:



The true shear stress is:

$$\tau_W = \frac{\Delta P}{2\left(\frac{L}{R} + e\right)}$$

Summary of Corrections

Calculate the apparent shear rate: $$\dot{\gamma}_{app} = \frac{4Q}{\pi R^3}$$

Correct the shear rate by using the Rabinowitch correction:

$$\dot{\gamma}_W = \dot{\gamma}_{app}\left(\frac{3}{4} + \frac{1}{4}\frac{d\ln Q}{d\ln \tau_W}\right)$$

Obtain true shear stress by using Bagley correction:

$$\tau_W = \frac{\Delta P}{2\left(\frac{L}{R} + e\right)}$$

Calculate true viscosity: $$\eta = \frac{\tau_w}{\dot{\gamma}_w}$$