FUNDAMENTALS OF POLYMER PROCESSING

STANLEY MIDDLEMAN

Professor of Chemical Engineering
and
Polymer Science and Engineering
University of Massachusetts, Amherst

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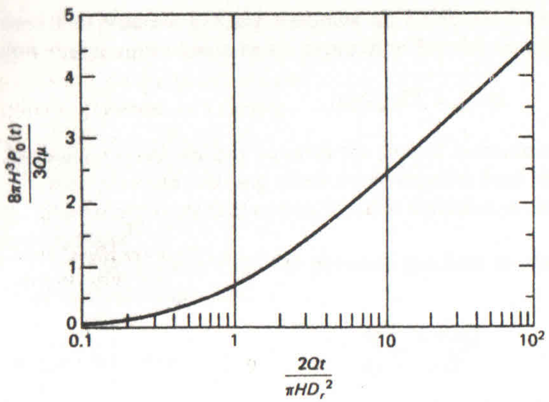

Figure 11-11 Pressure rise: newtonian isothermal filling of a disk at constant flow rate.

or

$$R^{*2} = \frac{Q}{2\pi H} t + \tfrac{1}{4}D_r^2 \tag{11-27}$$

The pressure P_0 follows from Eq. (11-21) and increases with time according to

$$P_0 = \frac{3Q\mu}{8\pi H^3} \ln\left(1 + \frac{2Q}{\pi H D_r^2} t\right) \tag{11-28}$$

Figure 11-11 shows the pressure rise for this case.

Example 11-1 We will analyze the behavior of a simple disk mold, as illustrated in Fig. 11-8, choosing the following parameters:

$$L_r = 1 \text{ cm} \qquad R = 9 \text{ cm}$$
$$D_r = 1 \text{ cm} \qquad 2H = 0.316 \text{ cm}$$
$$\mu = 10^5 \text{ P} \qquad \rho = 1 \text{ g/cm}^3$$

Two cases will be considered:

(*a*) Constant Q, with a fill time specified to be 1 s.
(*b*) Constant P_0, with a fill time specified to be 1 s.

Case (a): Constant Q. From Eq. (11-25),

$$Q = \frac{2\pi H}{t^*}\left(R^2 - \frac{Dr^2}{4}\right)$$

From Eq. (11-28),

$$P_0 = \frac{3Q\mu}{8\pi H^3} \ln\left(1 + \frac{2Q}{\pi H D_r^2} t\right)$$

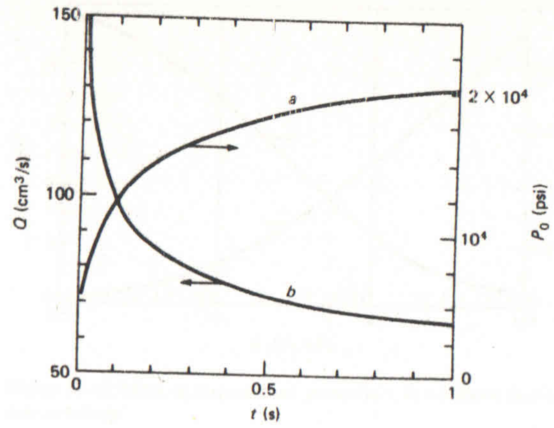

Figure 11-12 Solutions of Example 11-1: (*a*) $P_0(t)$ at constant Q; (*b*) $Q(t)$ at constant P_0.

With the parameters given we find the pressure buildup illustrated in Fig. 11-12*a*.

Let us examine the pressure drop to be expected *in the runner* under these conditions. From Poiseuille's law we find

$$\Delta P_r = \frac{128\mu L_r Q}{\pi D_r^4}$$

which gives a pressure drop of about 5000 psi. This constant pressure must be added to P_0 in order to overcome the resistance of the runner. It is not difficult to see that a runner of much smaller diameter than 1 cm will give a pressure drop quite a bit larger than that associated with the cavity flow itself, for this particular cavity.

Case (b): Constant P_0. Here we use Fig. 11-10. For $a = 0.055$ we find $Bt^* = 1.2$. Since we want $t^* = 1$ s, this gives

$$B = \frac{H^2 P_0}{3\mu R^2} = 1.2$$

from which it follows that $P_0 = 17{,}200$ psi. Figure 11-12*b* shows $Q(t)$ for this case, using Fig. 11-9.

11-2 AN EVALUATION OF VISCOUS HEATING IN A RUNNER

We shall find that the flow in the runner is often at such high shear rates that significant viscous heating is generated. Since the temperature dependence of viscosity is so strong, this factor must normally be accounted for. The simplest model of viscous heating assumes an adiabatic flow, for which the first law of thermodynamics leads to an expression for the temperature rise in the form

$$\rho Q C_p \, dT = Q \, dp \tag{11-29}$$

Equation (11-29) simply equates the rate of increase of thermal energy to the rate at which pressure is doing work in moving the fluid through the runner at the rate Q. The temperature that appears in this equation is the *flow average*, or *cup-mixing* temperature.

We will assume that the pressure *gradient* in the runner is given by a local form of Poiseuille's law:

$$\frac{dp}{dz} = \frac{128\mu Q}{\pi D^4} \tag{11-30}$$

where the viscosity μ may be a function of z. We will take the viscosity to depend on temperature according to

$$\frac{\mu}{\mu_0} = e^{-b(T-T_0)} \tag{11-31}$$

In the subsequent analysis we take T_0 to be the melt temperature at the entrance to the runner, and μ_0 to be the viscosity at that temperature.

If the pressure gradient is replaced by the corresponding function of temperature, using the last two equations, we obtain a differential equation for $T(z)$ of the form

$$\frac{dT}{dz} = \frac{\Delta P_0}{\rho C_p L} e^{-b(T-T_0)} \tag{11-32}$$

where ΔP_0 is the pressure drop that would exist under isothermal conditions at $T = T_0$:

$$\Delta P_0 = \frac{128\mu_0 L Q}{\pi D^4} \tag{11-33}$$

Equation (11-32) is easily integrated with the result

$$\chi = 1 + \frac{b \, \Delta P_0 \, z}{\rho C_p \, L} \tag{11-34}$$

where

$$\chi = e^{b(T-T_0)} \tag{11-35}$$

(Recall the appearance of χ in Chap. 6 in the analysis of adiabatic extrusion.) The temperature rise over the length L is given by

$$\chi_L = 1 + \frac{b \, \Delta P_0}{\rho C_p} \tag{11-36}$$

Figure 11-13 shows this result in a simple dimensionless format.

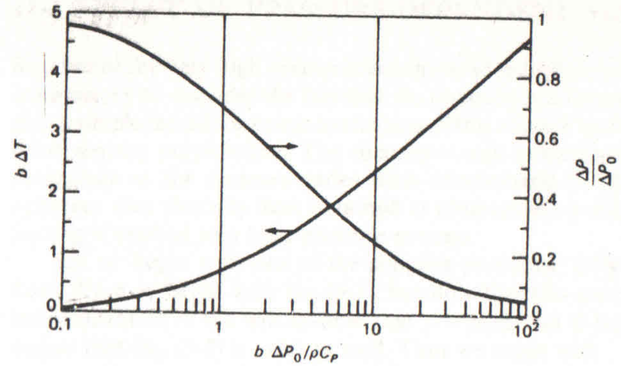

Figure 11-13 Effect of viscous heat generation, in adiabatic capillary flow, on temperature rise and pressure drop.

To find the actual pressure drop we must go back to Eq. (11-30), substitute $\mu[T(z)]$, and integrate. The result is easily found and is conveniently expressed in the form

$$\frac{\Delta P}{\Delta P_0} = \frac{\ln \chi_L}{\chi_L - 1} \tag{11-37}$$

Figure 11-13 shows this result.

Example 11-2 An ABS melt is being injection molded and enters a circular cross-sectional runner of diameter 0.4 in and length 4 in at an inlet melt temperature of 415°F. Estimate the pressure drop and the outlet temperature as a function of injection rate in the range $0.5 < Q < 20$ in^3/s.

Physical properties for the ABS polymer, at the inlet temperature, are

$$\rho = 1.12 \text{ g/cm}^3 \qquad n = \tfrac{1}{3}$$
$$C_p = 0.4 \text{ cal/g}\cdot\text{K} \qquad K = 2.6 \times 10^5 \text{ dyne}\cdot\text{s}^{1/3}\text{ cm}^2$$
$$b = 0.026 \text{ K}^{-1}$$

We will solve the problem using the *newtonian adiabatic* analysis with a viscosity appropriate to each injection rate.

We will need the following results established earlier in Chap. 5:

Nominal shear rate: $\dot\gamma = \dfrac{8(1+3n)Q}{n\pi D^3}$

Poiseuille's law: $\Delta P_0 = \dfrac{128 QL}{\pi D^4}\dfrac{3n+1}{4n}\eta$

Power law: $\eta = K\dot\gamma^{n-1}$