STREAMLINES AND STREAMFUNCTION

A *streamline* is the locus of points that are everywhere tangent to the instantaneous velocity vector **v**. If **ds** is an element of length along a streamline, and thus tangent to the local velocity vector, then the equation of a streamline is given by (Fig. 1)

$$\boldsymbol{ds} \times \boldsymbol{v} = 0 \qquad (1)$$

or, in 2D Cartesian coordinates

$$\frac{dx}{u} = \frac{dy}{v} \qquad (2)$$

Two streamlines cannot intersect except where **v**=0

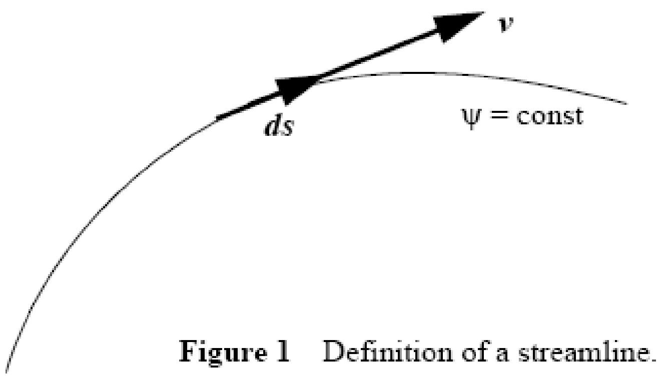


**Figure 1** Definition of a streamline.

Since, by definition, no flow can cross a streamline it requires that the velocity vector field **v** have to be divergence-free (solenoidal)

$$\nabla \cdot \boldsymbol{v} = 0 \qquad (3)$$

That means the flow is to be steady-state and no distributed sources and sinks can exist in the flow domain.
An equation that would describe such streamlines in a 2D (and axisymmetric) flow may be written in the form

$$\psi = \psi(x, y) \qquad (4)$$

Where **Ψ** is called the streamfunction. When **Ψ** is constant (4) describes a streamline. It must obey the general differential relation for the change in the streamfunction, d**Ψ**,

$$d\psi = \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy \qquad (5)$$

The following definition relates $\Psi$ and the velocity components

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

The definition (6) automatically satisfies the condition of free divergence (3). Substituting (6) into (5) it gives

$$d\psi = -v\ dx + u\ dy \tag{7}$$

A major characteristic of the streamfunction is that the difference $\Psi$ in between two streamlines is equal to the volume flow rate Q between those streamlines. Let us consider two streamlines with values $\Psi_A$ and $\Psi_B$ as shown in Fig. 2.

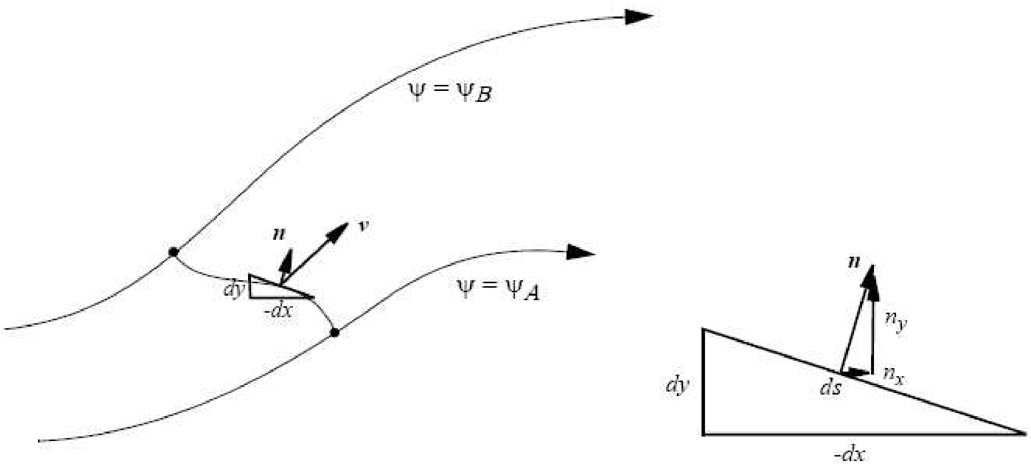


**Figure 2** Streamfunction in a plane flow.

The volume flow rate between the streamlines is

$$Q_{AB} = \int_A^B \boldsymbol{v} \cdot \boldsymbol{n}\ ds = \int_A^B (un_x + vn_y)ds \tag{8}$$

Where **n** is the normal unit vector. By geometry we have the relations $n_x ds = dy$ and $n_y ds = -dx$.With these relations (8) becomes

$$Q_{AB} = \int_A^B (udy - vdx) = \int_A^B d\psi$$

$$Q_{AB} = \psi_B - \psi_A \tag{9}$$

The flow rate between streamlines is the difference in their streamfunction values. This equation is also unaffected by the addition of an arbitrary constant to $\Psi$.